Estimating Conductivities and Dipole Source Signal with EEG Arrays

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Abstract

Techniques based on electroencephalography (EEG) measure electric potentials on the scalp and process them to infer the location, distribution, and intensity of the underlying neural activity. The accuracy in estimating these parameters is highly sensitive to the uncertainty in the conductivities of the head tissues. Furthermore, dissimilarities among individuals are ignored when standard values obtained by direct measurements of in vivo and in vitro tissue samples are used. Methods such as electrical impedance tomography (EIT) enable individual estimation, but they require each patient to be subject of a study for estimating his/her tissues' conductivities before, and in addition to, the EEG measurements. Recently, magnetic resonance diffusion-weighted imaging techniques have been developed to estimate conductivities on an individual basis. Here, we present statistical processing methods that allow simultaneous estimation of the layer conductivity ratios and the source signal using EEG data. We use the classical 4-sphere model to approximate the head geometry, and assume known dipole source position. We apply the maximum likelihood (ML) technique and Bayesian approach to obtain, respectively, the ML and maximum a posteriori (MAP) estimates of the conductivities and dipole moment. The accuracy of our estimates is evaluated by comparing their variances with the corresponding Cramér-Rao bound (CRB). We show that the proposed method provides estimates with variances close to the CRB for sufficiently large data. The results are illustrated by a numerical example using different signal-to-noise ratio (SNR) values.

1. Introduction

The problem of dipole source localization and signal estimation is of great interest in neuroscience. It has applications in areas such as clinical sciences and brain research. Techniques based on electroencephalography (EEG) measure the electric potentials on the scalp and process them to infer the location and signal of the underlying neural activity. It has been shown that the accuracy of estimating these source parameters is highly sensitive to the uncertainty in the conductivities of most of the head tissues [1]. These conductivities are typically obtained by direct measurements of *in vivo* and *in vitro* samples of the tissues involved [2]. Then, dissimilarities in the conductivities among individuals are not considered. Other methods like impedance tomography [3] and magnetic resonance using current density imaging [4] allow individual estimation, but they require each patient to be a subject of a study for estimating his/her tissues' conductivities before, and in addition to, the EEG measurements. Recently, magnetic resonance diffusion-weighted imaging techniques have been developed to estimate conductivities on an individual basis [5], and simultaneous magnetoencephalography (MEG) and EEG analysis has been used to derive equivalent conductivity estimates that improve the estimation of dipole source parameters [6].

In this paper, we develop statistical methods that allow simultaneous estimation of the ratios of the layer conductivities and source signal using EEG array data. We assume the classical concentric 4-sphere model to approximate the head geometry, and an array of EEG sensors to measure the electric potential on the scalp. We also assume that the geometry of the head's layers and the position of the source are known. The last assumption holds in practice for evoked response and event-related experiments. In these cases, the response is approximately a predictable and repetitive equivalent dipole with known location.

We propose an estimation method based on the maximum likelihood (ML) technique, which is asymptotically efficient under general "regularity" conditions [7]. For cases when an approximate description of the parameters is given by a probabilistic distribution, we consider the maximum a-posteriori (MAP) estimate, which is obtained using the Bayesian approach under the consideration of random parameters with known a-priori distribution. Furthermore, we compute the Cramér-Rao bounds (CRB) for our estimates. The CRB is useful as it provides a universal reference for evaluating the performance of unbiased estimates [8]. Finally, numerical examples demonstrate the applicability of our methods to a practical EEG measurement system.

2. The Proposed Methods

Consider the forward problem of computing the surface potential for a dipole source. We assume that the geometry of the head and the location of the source are known.

For the head model, we choose a multishell spherical model which includes four concentric layers for the brain, cerebrospinal fluid (CSF), skull, and scalp. These layers are assumed to be isotropic and to have homogeneous conductivities $\sigma_1, \ldots, \sigma_4$, and radii $\rho_1, \ldots, \rho_4$, respectively.

For the source model, consider a single dipole with a moment $\tilde{\boldsymbol{q}} = [\tilde{q}_x, \tilde{q}_y, \tilde{q}_z]^T$ located at a point $\boldsymbol{p} = [p_x, p_y, p_z]^T$ in the brain. The surface potential at the ith sensor located on the scalp at $\boldsymbol{r}_i = [r_{ix}, r_{iy}, r_{iz}]^T$, $i = 1, \ldots, m$, where m is the number of electrodes, can be expressed as $v_i = \boldsymbol{g}_i^T(\boldsymbol{\theta})\boldsymbol{q}$, where $\boldsymbol{q} = \tilde{\boldsymbol{q}}/\sigma_4$, $\boldsymbol{g}_i(\boldsymbol{\theta})$ is the gain vector (or "field kernel"), and $\boldsymbol{\theta} = [\theta_1, \theta_2, \theta_3]^T$ the unknown parameter vector defined by the ratios of the conductivities, with $\theta_j = \sigma_j/\sigma_{j+1}$, $j = 1, \ldots, 3$.

Under the above conditions, we can express $\boldsymbol{g}_i$ for our 4-layer model as

$$\boldsymbol{g}_i(\boldsymbol{\theta}) = \sum_{k=1}^{n} w \left(\frac{\|\boldsymbol{p}\|}{\rho_4}\right)^{k-1} \left[\boldsymbol{u}_x P_k^1(\cos\vartheta_i)\cos\varphi_i + \boldsymbol{u}_y P_k^1(\cos\vartheta_i)\sin\varphi_i + \boldsymbol{u}_z k P_k(\cos\vartheta_i)\right] \quad (1)$$

where n represents the number of terms in the expansion (which ideally should be infinite); $P_k(\cdot)$ is the Legendre polynomial of order k; $P_k^1(\cdot)$ is the associated Legendre Polynomial; $\boldsymbol{u}_x$, $\boldsymbol{u}_y$, and $\boldsymbol{u}_z$ are the unitary vectors in the x, y, and z directions, respectively; ϑ_i, φ_i are the azimuth and elevation angles of the position vector $\boldsymbol{r}_i$ (see Fig. 1); the weighting function $w = w(\boldsymbol{\theta}; k; \rho_1, \ldots, \rho_4)$ is defined in [9]. Note that, without loss of generality, we have implicitly assumed in equation (1) that the dipole is located on the z-axis.

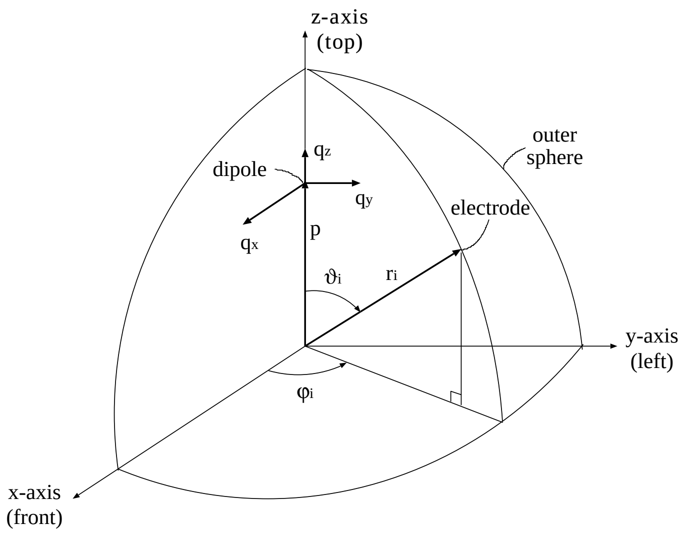

Figure 1: Dipole in a spherical head model.

Define the array response as a matrix $A(\boldsymbol{\theta})$ of size $m \times 3$ where the ith row corresponds to $\boldsymbol{g}_i^T$. Then, allowing $\boldsymbol{q}$ to change in time and assuming that the source remains in the same position during the measurements period, we have that $\boldsymbol{v}(t) = A(\boldsymbol{\theta})\boldsymbol{q}(t)$, which is a *spatio-temporal* representation of the potential vector $\boldsymbol{v} = [v_1, \ldots, v_m]^T$ measured by the m electrodes.

2.1. Estimating the Conductivities

We wish to estimate the unknown conductivity parameters $\boldsymbol{\theta}$. Let $\boldsymbol{y}(t)$ denote the $m \times 1$ measurement vector obtained from the EEG sensors at time t. Assume that the measurements are in discrete time, and are taken in the presence of zero mean Gaussian noise $\boldsymbol{e}(t)$ uncorrelated between samples. Then, the measurement model is given by

$$\boldsymbol{y}(t) = A(\boldsymbol{\theta})\boldsymbol{q}(t) + \boldsymbol{e}(t) \ , \qquad t = 1, \ldots, N. \quad (2)$$

where N is the number of time samples. Observe that it is possible to estimate only the conductivity ratios $\boldsymbol{\theta}$ and moments $\boldsymbol{q}(t)$. Simultaneous estimation of $[\sigma_1, \sigma_2, \sigma_3, \sigma_4]^T$ and $\tilde{\boldsymbol{q}}(t)$ is not possible since it would result in a non-identifiability problem [10]. Thus, we will refer to $\boldsymbol{\theta}$ and $\boldsymbol{q}(t)$ as the conductivities and dipole moment, respectively.

2.1.1. Maximum Likelihood Estimate

The problem of estimating $\boldsymbol{\theta}$ from (2) can be seen as that of estimating deterministic parameters from a Gaussian model. The ML estimate (MLE) is given by the value of $\boldsymbol{\theta}$ that minimizes the concentrated likelihood function (CLF) $f(\theta)$, defined in [11] as

$$f(\boldsymbol{\theta}) = \operatorname{tr}\left\{\left(I - \hat{A}(\hat{A}^T\hat{A})^{-1}\hat{A}^T\right)\hat{R}\right\} \quad (3)$$

where $\hat{A}$ denotes the MLE of A (i.e., $\hat{A} = A(\hat{\boldsymbol{\theta}})$), $\operatorname{tr}\{\cdot\}$ is the trace operator, and $\hat{R}$ is a consistent estimate of the covariance matrix of the observation vector, defined as $\hat{R} = \frac{1}{N}\sum_{t=1}^{N} \boldsymbol{y}(t)\boldsymbol{y}(t)^T$. Once $\hat{A}$ has been calculated, the MLE of the dipole moment can be obtained by a simple least-squares fit, i.e. $\hat{\boldsymbol{q}}(t) = \left(\hat{A}^T\hat{A}\right)^{-1}\hat{A}^T\boldsymbol{y}(t)$.

2.1.2. Bayesian Approach

Consider $\boldsymbol{\theta}$ as a random parameter with a known a-priori distribution $h(\boldsymbol{\theta})$. We can define the maximum a-posteriori (MAP) estimate as the value of $\boldsymbol{\theta}$ that minimizes the following log-likelihood function

$$J(\boldsymbol{\theta}) = mN \ln f(\boldsymbol{\theta}) + \ln h(\boldsymbol{\theta}) \quad (4)$$

where the first term corresponds to the value of the sampling distribution of $\boldsymbol{y}$ for a fixed value of $\boldsymbol{\theta}$, and the second describes the a-priori knowledge on the distribution of $\boldsymbol{\theta}$.

3. Numerical Examples

We conducted a series of simulations for EEG measurements in the 4-layer spherical head model previously described. The nominal radii of the 4 layers corresponding to the brain, CSF, skull, and scalp, were chosen to be $[\rho_1, \rho_2, \rho_3, \rho_4] = [7.9, 8.1, 8.5, 8.8]$ cm.

To simulate our source, we chose a current dipole located at $\boldsymbol{p} = [0, -5.6, 5.6]^T$ cm. The components $\tilde{q}_x$, $\tilde{q}_y$, and $\tilde{q}_z$ of the dipole change in time according to

$$\begin{aligned}
\tilde{q}_x(t) &= 3e^{-(t-40)^2/17^2} - 13e^{-(t-60)^2/12^2}[\mathrm{nA}\cdot\mathrm{m}],\\
\tilde{q}_y(t) &= 0 \ ,\\
\tilde{q}_z(t) &= 15e^{-(t-60)^2/8^2} - 5e^{-(t-40)^2/17^2}\ [\mathrm{nA}\cdot\mathrm{m}].
\end{aligned} \quad (5)$$

Note that in this set of equations t is continuous and with units of milliseconds. We sampled these signals at a rate of 0.1 ms thus obtaining $N = 1000$ samples for our computer simulations.

We added uncorrelated (in time and space) random noise, distributed as $\mathcal{N}(0, \sigma^2)$. For the conductivities, we used $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [0.33, 1, 0.0042, 0.33]$ $(\Omega\mathrm{m})^{-1}$, which sets the real value of our parameter as $\boldsymbol{\theta} = [0.33, 238.095, 0.01273]^T$. For the measurements, we used an EEG configuration of $m = 61$ electrodes located on a sphere of radius $\rho_4 = 8.8$ cm with a single sensor at the top position and 4 rings at elevation angles of $\pi/12$, $\pi/6$, $\pi/4$, and $\pi/3$ rad, containing, respectively 6, 12, 18, and 24 sensors equally spaced in the azimuthal direction. This arrangement is similar to an array made commercially by Neurosoft, Inc. (http:\\www.neuro.com).

Under the conditions previously described and the model (2), we generated our numerical data. Then, we determined the ML estimates for the conductivities by minimizing the corresponding CLF. We initialized this minimization process with $\boldsymbol{\theta} = [0.25, 200, 0.01]^T$.

In order to analyze the behavior of the MLE under different signal-to-noise ratio (SNR) values, we varied the noise variance σ^2 using $n = 80$ terms in equation (1). We defined the SNR in dB as $\text{SNR} = 20\log(\bar{\boldsymbol{v}}^T\bar{\boldsymbol{v}}/\sigma^2)$, with $\bar{v}_i = \frac{1}{N}\sum_{t=1}^{N} v_i(t)$, and $\boldsymbol{v}(t)$ as previously defined.

For the estimation process, we repeated the experiment 50 times with independent noise realizations. Then, we compared the variances of the estimates (denoted as $\sigma^2_{\theta_j}$) with the corresponding CRB. The resulting variance curves are shown in Fig. 2. Note that even for low SNR, the variances of our estimates are close to the CRB. In fact, the values of $\sigma^2_{\theta_1}$ and $\sigma^2_{\theta_3}$ are below the CRB at some points, which reflects the fact of having biased estimates with low variance. However, other experiments (to be included in the full version of this article) have also shown that our estimates can have a bias below 6% for SNR values over 55 dB and a sufficient number of terms n used in the calculation of the surface potentials.

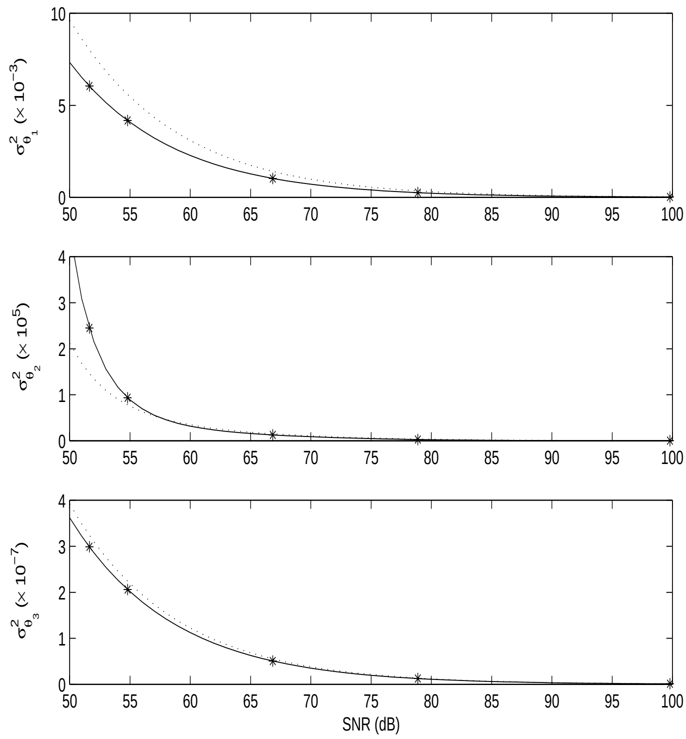

Figure 2: Variances of the conductivity estimates (full line) compared with the Cramér-Rao bound (dashed line).

4. Concluding Remarks

We have applied the ML method to estimate the ratios of the conductivities of the different layers in the brain using EEG array measurements. We assumed a spherical head model and known dipole position.

Our results show that is possible to obtain asymptotically consistent estimates with variances that approach the Cramér-Rao bound for a sufficiently large number of terms n used from the infinite sum in equation (1).

Our method has the potential of improving the accuracy of dipole estimates in practical cases, since the conductivities are usually unknown and vary among individuals. In addition, this method has an easy implementation and allows a simultaneous estimation of the dipole moments, reducing the time required between experiments.

Other developments and experiments relevant to applying our method to calculate the ML and MAP estimates, CRB and sensitivity analysis will be available in the full version of this article. Further research in this area will include more extensive applications to realistic head modeling using numerical solutions such as BEM or FEM. We will also develop techniques for simultaneous estimation of conductivities, dipole moment, and position.

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